MAT A

MATEMATIKA

viša razina

KNJIŽICA FORMULA

Matematika

Knjižica formula

F O R M U L E

- Kompleksan broj: $i^2 = -1,\ z = a + bi,\ \bar{z} = a - bi,\ |z| = \sqrt{a^2 + b^2},\ a, b \in \mathbf{R}$

- $z = r(\cos\varphi + i\sin\varphi),\ z_1 \cdot z_2 = r_1 r_2 (\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2)),$

  $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}(\cos(\varphi_1 - \varphi_2) + i\sin(\varphi_1 - \varphi_2)),\ z^n = r^n(\cos n\varphi + i\sin n\varphi),$

  $\sqrt[n]{z} = \sqrt[n]{r}\left(\cos\left(\dfrac{\varphi + 2k\pi}{n}\right) + i\sin\left(\dfrac{\varphi + 2k\pi}{n}\right)\right),\quad k = 0, 1, \ldots, n-1$

- $a^m \cdot a^n = a^{m+n},\quad a^m : a^n = a^{m-n}\ (a \neq 0),\quad a^{-m} = \dfrac{1}{a^m}\ (a \neq 0),\quad \sqrt[m]{a^n} = a^{\frac{n}{m}}$

- $(a \pm b)^2 = a^2 \pm 2ab + b^2,\qquad\qquad (a \pm b)^3 = a^3 \pm 3a^2 b + 3ab^2 \pm b^3$

- $a^2 - b^2 = (a - b)(a + b),\qquad\qquad a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

- $(a + b)^n = a^n + \dbinom{n}{1}a^{n-1}b + \ldots + \dbinom{n}{k}a^{n-k}b^k + \ldots + \dbinom{n}{n-1}ab^{n-1} + b^n$

- Kvadratna jednadžba: $ax^2 + bx + c = 0, a \neq 0 \Rightarrow x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- Vièteove formule: $x_1 + x_2 = -\dfrac{b}{a},\ x_1 \cdot x_2 = \dfrac{c}{a}$

- Tjeme parabole: $T\left(-\dfrac{b}{2a}, \dfrac{4ac - b^2}{4a}\right)$

- $b^x = a \Leftrightarrow x = \log_b a,\qquad\qquad \log_b b^x = x = b^{\log_b x}$

- $\log_b(xy) = \log_b x + \log_b y,\quad \log_b \dfrac{x}{y} = \log_b x - \log_b y,\quad \log_b x^y = y\log_b x,\quad \log_a x = \dfrac{\log_b x}{\log_b a}$

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- Površina trokuta: $P = \dfrac{a \cdot v_a}{2}, \quad P = \sqrt{s \cdot (s-a) \cdot (s-b) \cdot (s-c)}, \quad s = \dfrac{a+b+c}{2}$

$$P = \frac{ab \sin \gamma}{2}, \quad P = \frac{abc}{4r_o}, \quad P = r_u s$$

- Jednakostraničan trokut: $P = \dfrac{a^2 \sqrt{3}}{4}, \quad v = \dfrac{a\sqrt{3}}{2}, \quad r_o = \dfrac{2}{3}v, \quad r_u = \dfrac{1}{3}v$

- Površina paralelograma: $P = a \cdot v$

- Površina trapeza: $P = \dfrac{a+c}{2} \cdot v$

- Površina kruga: $P = r^2 \pi$

- Opseg kruga: $O = 2r\pi$

- Površina kružnoga isječka: $P = \dfrac{r^2 \pi \alpha}{360}$

- Duljina kružnoga luka: $l = \dfrac{r \pi \alpha}{180}$

B = površina osnovke (baze), $\quad P$ = površina pobočja, $\quad h$ = duljina visine

- Obujam (volumen) prizme i valjka: $V = B \cdot h$

- Oplošje prizme i valjka: $O = 2B + P$

- Obujam (volumen) piramide i stošca: $V = \dfrac{1}{3} B \cdot h$

- Oplošje piramide: $O = B + P$

- Oplošje stošca: $O = r^2 \pi + r \pi s$

  r = polumjer osnovke $\quad s$ = duljina izvodnice

- Obujam (volumen) kugle: $V = \dfrac{4}{3} r^3 \pi$

- Oplošje kugle: $O = 4r^2 \pi$, r = polumjer kugle

- U pravokutnome trokutu:

$$\text{sinus kuta} = \frac{\text{nasuprotna kateta}}{\text{hipotenuza}}, \quad \text{kosinus kuta} = \frac{\text{priležeća kateta}}{\text{hipotenuza}},$$

$$\text{tangens kuta} = \frac{\text{nasuprotna kateta}}{\text{priležeća kateta}}$$

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- Poučak o sinusima: $\dfrac{a}{\sin \alpha} = \dfrac{b}{\sin \beta} = \dfrac{c}{\sin \gamma}$
- Poučak o kosinusima: $c^2 = a^2 + b^2 - 2ab\cos \gamma$

- $\sin^2 x + \cos^2 x = 1, \qquad \text{tg}\, x = \dfrac{\sin x}{\cos x}$

- $\sin 2x = 2\sin x \cos x, \qquad \cos 2x = \cos^2 x - \sin^2 x$

- $\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$

  $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

  $\text{tg}(x \pm y) = \dfrac{\text{tg}\, x \pm \text{tg}\, y}{1 \mp \text{tg}\, x \cdot \text{tg}\, y}$

- $\sin x + \sin y = 2\sin \dfrac{x+y}{2}\cos \dfrac{x-y}{2}, \qquad \sin x - \sin y = 2\cos \dfrac{x+y}{2}\sin \dfrac{x-y}{2}$

  $\cos x + \cos y = 2\cos \dfrac{x+y}{2}\cos \dfrac{x-y}{2}, \qquad \cos x - \cos y = -2\sin \dfrac{x+y}{2}\sin \dfrac{x-y}{2}$

- $\sin x \sin y = \dfrac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$

  $\cos x \cos y = \dfrac{1}{2}\left[\cos(x-y) + \cos(x+y)\right]$

  $\sin x \cos y = \dfrac{1}{2}\left[\sin(x-y) + \sin(x+y)\right]$

- $\sin \dfrac{\pi}{6} = \dfrac{1}{2}, \qquad\qquad \sin \dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}, \qquad\qquad \sin \dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$

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- Udaljenost točaka T_1, T_2 : $d(T_1, T_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

- Polovište dužine $\overline{T_1 T_2}$: $P\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

- Vektor $\overrightarrow{T_1 T_2}$: $\overrightarrow{T_1 T_2} = \vec{a} = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} = a_1 \vec{i} + a_2 \vec{j}$

- Skalarni umnožak vektora: $\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\alpha, \quad \vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2$

- Jednadžba pravca: $y - y_1 = k(x - x_1), \quad k = \dfrac{y_2 - y_1}{x_2 - x_1}$

- Kut α između dvaju pravaca: $\operatorname{tg}\alpha = \left|\dfrac{k_2 - k_1}{1 + k_1 k_2}\right|$

- Udaljenost točke $T(x_1, y_1)$ i pravca $p\ldots$ $Ax + By + C = 0$: $d(T, p) = \dfrac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

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Krivulja drugoga reda	Jednadžba	Tangenta u točki krivulje (x_1, y_1)
Kružnica središte $S(p,q)$	$(x-p)^2 + (y-q)^2 = r^2$	$(x_1-p)(x-p) + (y_1-q)(y-q) = r^2$
Elipsa fokusi $F_{1,2}(\pm e, 0)$ $e^2 = a^2 - b^2$	$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$	$\dfrac{x_1 x}{a^2} + \dfrac{y_1 y}{b^2} = 1$
Hiperbola fokusi $F_{1,2}(\pm e, 0)$ $e^2 = a^2 + b^2$ asimptote $y = \pm \dfrac{b}{a} x$	$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$	$\dfrac{x_1 x}{a^2} - \dfrac{y_1 y}{b^2} = 1$
Parabola fokus $F\left(\dfrac{p}{2}, 0\right)$ direktrisa $x = -\dfrac{p}{2}$	$y^2 = 2px$	$y_1 y = p(x + x_1)$

- Uvjet dodira pravca $y = kx + l$ i kružnice: $r^2(1 + k^2) = (kp - q + l)^2$

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- Aritmetički niz: $a_n = a_1 + (n-1)\cdot d,$ $S_n = \dfrac{n}{2}(a_1 + a_n)$

- Geometrijski niz: $a_n = a_1 \cdot q^{n-1},$ $S_n = a_1 \dfrac{q^n - 1}{q-1}$

- Geometrijski red: $S = \dfrac{a_1}{1-q},$ $|q| < 1$

- Derivacija umnoška: $(f \cdot g)' = f' \cdot g + f \cdot g'$ • Derivacija kvocijenta: $\left(\dfrac{f}{g}\right)' = \dfrac{f' \cdot g - f \cdot g'}{g^2}$

- Derivacija kompozicije: $(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$

- Tangenta na graf funkcije f u $T(x_1, y_1)$: $y - y_1 = f'(x_1) \cdot (x - x_1)$

- Derivacije:

$c' = 0$	$(x^n)' = n \cdot x^{n-1}, n \neq 0$	$(\sin x)' = \cos x$	$(\cos x)' = -\sin x$	$(\operatorname{tg} x)' = \dfrac{1}{\cos^2 x}$

MAT T A

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Prazna stranica