



Nacionalni centar
za vanjsko vrednovanje
obrazovanja

Identifikacijska
naljepnica

PAŽLJIVO NALIJEPI TI

MAT A

MATEMATIKA

viša razina

KNJIŽICA FORMULA

MAT T A

MATA.41.HR.R.T2.08



30439



12

Matematika

Knjižica formula

FORMULE

- Standardni zapis kompleksnog broja: $z = a + bi$, $a, b \in \mathbb{R}$, $i^2 = -1$, $\bar{z} = a - bi$, $|z| = \sqrt{a^2 + b^2}$

- Trigonometrijski zapis kompleksnog broja: $z = r(\cos \varphi + i \sin \varphi)$, $\varphi \in [0, 2\pi)$,

$$z_1 \cdot z_2 = r_1 r_2 (\cos(\varphi_1 + \varphi_2) + i \sin(\varphi_1 + \varphi_2))$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} (\cos(\varphi_1 - \varphi_2) + i \sin(\varphi_1 - \varphi_2)), \quad z^n = r^n (\cos n\varphi + i \sin n\varphi),$$

$$\sqrt[n]{z} = \sqrt[n]{r} \left(\cos \left(\frac{\varphi + 2k\pi}{n} \right) + i \sin \left(\frac{\varphi + 2k\pi}{n} \right) \right), \quad k = 0, 1, \dots, n-1$$

- $a^m \cdot a^n = a^{m+n}$, $a^m : a^n = a^{m-n}$ ($a \neq 0$), $a^{-m} = \frac{1}{a^m}$ ($a \neq 0$), $\sqrt[n]{a^n} = a^{\frac{n}{n}}$

- $(a \pm b)^2 = a^2 \pm 2ab + b^2$, $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

- $a^2 - b^2 = (a - b)(a + b)$, $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

- $(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \dots + \binom{n}{k} a^{n-k} b^k + \dots + \binom{n}{n-1} a b^{n-1} + b^n$, $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

- Kvadratna jednačina: $ax^2 + bx + c = 0$, $a \neq 0 \Rightarrow x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- Vièteove formule: $x_1 + x_2 = -\frac{b}{a}$, $x_1 \cdot x_2 = \frac{c}{a}$

- Tjeme parabole: $T \left(-\frac{b}{2a}, \frac{4ac - b^2}{4a} \right)$

- $b^x = a \Leftrightarrow x = \log_b a$, $\log_b b^x = x = b^{\log_b x}$

- $\log_b(xy) = \log_b x + \log_b y$, $\log_b \frac{x}{y} = \log_b x - \log_b y$, $\log_b x^y = y \log_b x$, $\log_a x = \frac{\log_b x}{\log_b a}$



Matematika

Knjižica formula

- Površina trokuta: $P = \frac{a \cdot v_a}{2}$, $P = \sqrt{s \cdot (s-a) \cdot (s-b) \cdot (s-c)}$, $s = \frac{a+b+c}{2}$

$$P = \frac{ab \sin \gamma}{2}, \quad P = \frac{abc}{4r_o}, \quad P = r_u s$$

- Jednakostraničan trokut: $P = \frac{a^2 \sqrt{3}}{4}$, $v = \frac{a \sqrt{3}}{2}$, $r_o = \frac{2}{3} v$, $r_u = \frac{1}{3} v$

- Površina paralelograma: $P = a \cdot v$

- Površina trapeza: $P = \frac{a+c}{2} \cdot v$

- Površina kruga: $P = r^2 \pi$

- Opseg kruga: $O = 2r\pi$

- Površina kružnoga isječka: $P = \frac{r^2 \pi \alpha}{360}$

- Duljina kružnoga luka: $l = \frac{r \pi \alpha}{180}$

B = površina osnovke (baze), P = površina pobočja, h = duljina visine

- Obujam (volumen) prizme i valjka: $V = B \cdot h$

- Oplošje prizme i valjka: $O = 2B + P$

- Obujam (volumen) piramide i stošca: $V = \frac{1}{3} B \cdot h$

- Oplošje piramide: $O = B + P$

- Oplošje stošca: $O = r^2 \pi + r \pi s$

r = polumjer osnovke s = duljina izvodnice

- Obujam (volumen) kugle: $V = \frac{4}{3} r^3 \pi$

- Oplošje kugle: $O = 4r^2 \pi$, r = polumjer kugle

- U pravokutnome trokutu:

$$\sinus \text{ kuta} = \frac{\text{duljina nasuprotne katete}}{\text{duljina hipotenuze}}, \quad \cosinus \text{ kuta} = \frac{\text{duljina priležeće katete}}{\text{duljina hipotenuze}},$$

$$\text{tangens kuta} = \frac{\text{duljina nasuprotne katete}}{\text{duljina priležeće katete}}$$

MAT T A



99

Matematika

Knjižica formula

- Poučak o sinusima: $\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$
- Poučak o kosinusima: $c^2 = a^2 + b^2 - 2ab \cos \gamma$

- $\sin^2 x + \cos^2 x = 1, \quad \operatorname{tg} x = \frac{\sin x}{\cos x}$

- $\sin 2x = 2 \sin x \cos x, \quad \cos 2x = \cos^2 x - \sin^2 x$

- $\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\operatorname{tg}(x \pm y) = \frac{\operatorname{tg} x \pm \operatorname{tg} y}{1 \mp \operatorname{tg} x \cdot \operatorname{tg} y}$$

- $\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}, \quad \sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}, \quad \cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$$

- $\sin x \sin y = \frac{1}{2} [\cos(x-y) - \cos(x+y)]$

$$\cos x \cos y = \frac{1}{2} [\cos(x-y) + \cos(x+y)]$$

$$\sin x \cos y = \frac{1}{2} [\sin(x-y) + \sin(x+y)]$$

- $\sin \frac{\pi}{6} = \frac{1}{2}, \quad \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}, \quad \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

MAT T A



99

Matematika

Knjižica formula

- Udaljenost točaka T_1, T_2 : $d(T_1, T_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Polovište dužine $\overline{T_1 T_2}$: $P\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- Vektor $\overrightarrow{T_1 T_2}$: $\overrightarrow{T_1 T_2} = \vec{a} = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} = a_1\vec{i} + a_2\vec{j}$
- Skalarni umnožak vektora: $\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos \alpha$, $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2$
- Jednadžba pravca: $y - y_1 = k(x - x_1)$, $k = \frac{y_2 - y_1}{x_2 - x_1}$
- Kut α između dvaju pravaca: $\operatorname{tg} \alpha = \left| \frac{k_2 - k_1}{1 + k_1 k_2} \right|$
- Udaljenost točke $T(x_1, y_1)$ i pravca $p \dots Ax + By + C = 0$: $d(T, p) = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$



Matematika

Knjižica formula

Krivulja drugoga reda	Jednadžba	Tangenta u točki krivulje (x_1, y_1)
Kružnica središte $S(p, q)$	$(x - p)^2 + (y - q)^2 = r^2$	$(x_1 - p)(x - p) + (y_1 - q)(y - q) = r^2$
Elipsa fokusi $F_{1,2}(\pm e, 0)$ $e^2 = a^2 - b^2$	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{x_1 x}{a^2} + \frac{y_1 y}{b^2} = 1$
Hiperbola fokusi $F_{1,2}(\pm e, 0)$ $e^2 = a^2 + b^2$ asimptote $y = \pm \frac{b}{a} x$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{x_1 x}{a^2} - \frac{y_1 y}{b^2} = 1$
Parabola fokus $F\left(\frac{p}{2}, 0\right)$ direktrisa $x = -\frac{p}{2}$	$y^2 = 2px$	$y_1 y = p(x + x_1)$

- Uvjet dodira pravca $y = kx + l$ i kružnice: $r^2(1 + k^2) = (kp - q + l)^2$



Matematika

Knjižica formula

- Aritmetički niz: $a_n = a_1 + (n-1) \cdot d, \quad S_n = \frac{n}{2}(a_1 + a_n)$

- Geometrijski niz: $a_n = a_1 \cdot q^{n-1}, \quad S_n = a_1 \frac{q^n - 1}{q - 1}$

- Geometrijski red: $S = \frac{a_1}{1 - q}, \quad |q| < 1$

- Derivacija umnoška: $(f \cdot g)' = f' \cdot g + f \cdot g'$ • Derivacija kvocijenta: $\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$

- Derivacija kompozicije: $(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$

- Tangenta na graf funkcije f u $T(x_1, y_1)$: $y - y_1 = f'(x_1) \cdot (x - x_1)$

- Derivacije:

$c' = 0$	$(x^n)' = n \cdot x^{n-1}, n \neq 0$	$(\sin x)' = \cos x$	$(\cos x)' = -\sin x$	$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$
----------	--------------------------------------	----------------------	-----------------------	---

MAT T A



99

Matematika

Prazna stranica

